The canonical form for congruence: some history and applications

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A square matrix A can be either seen as a linear map or as a bilinear form. When considered as a linear map, it is natural to introduce the relation of "similarity", $P^{-1}AP$, which is a change of basis that allows us to represent the linear map in a simpler form, in particular in the well-known "Jordan canonical form". When considering the matrix A as a bilinear form, the natural relation instead is the one of "congruence", $P^{\top}AP$, which is the suitable change of basis for bilinear forms. Is there a canonical form for such a relation? The answer is yes, and actually some different canonical forms for congruence have been introduced over the years since the 1930's. In this talk I will introduce the most recent one, review the history of all these canonical forms, and show some applications in the context of my past and current research, including:

- The solution of the equation $AX + X^{\top}A = 0$ and its connection to $\top$-palindromic pencils.
- The consistency of the equation $X^{\top}AX = B$ when B is either symmetric or skew-symmetric.